Prediction of Breakup Heights for Miscible Drops

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In CBE 30355 Transport Phenomena I at Notre Dame it has become a tradition to conduct in-class demonstrations of fluid mechanics phenomena. These demonstrations are chosen to elucidate concepts discussed in class and to excite student's interest in the subject. A visually appealing phenomenon is the fate of a miscible drop settling under the influence of gravity at low Reynolds numbers.

The fate of a miscible drop settling through a fluid at low Re is complex, and has been shown by Kojima, et al. (among others) to be divided into three phases. Under creeping flow conditions a spherical drop is neutrally stable even in the absence of surface tension. It will remain spherical, as the Stokes flow stresses on all parts of the surface are uniform. In practice, however, it will deform due to either the convection of any disturbances to the back of the sphere, eventually punching through and making a torus, or via inertial forces leading to the formation of an oblate spheroid which similarly progresses to a torus.

Once a torus has formed, the next stage of the evolution is expansion. If the torus is symmetric, then it will not expand under creeping flow conditions due to Stokes flow reversibility. Machu, et al., however, demonstrated that a torus composed of a dilute suspension of negatively buoyant particles in the same (viscous) fluid would expand if it were asymmetric in the flow (settling) direction. Inertial forces were demonstrated to lead to expansion by Kojima et al. In their work they used asymptotics to calculate the rate of expansion, and showed that it should be proportional to the Reynolds number. When compared to measurements, however, they found that a torus expands much more slowly than predicted. The discrepancy was attributed to a transient surface tension between the two miscible liquids arising from concentration gradients.

In the final stages of expansion the torus becomes unstable via a gravitational Rayleigh-Taylor mechanism and falls apart into two (or more) drops which then may form tori that become unstable in turn. This is the principal mechanism for drop breakup in the absence of surface tension or shear at low Re.

In recent years a number of investigators have looked at drops made up of dilute suspensions of particles. For small particles the drop velocity is much greater than the sedimentation velocity of the particles, and thus the cloud or blob behaves as if it were a miscible drop. Such drops have been studied both computationally and experimentally. The extensive experimental measurements of Pignatel et al., for dilute drop suspensions are particularly useful for comparison to the experimental measurements of Kojima et al for miscible drops.

In order to demonstrate the phenomena in a classroom it is necessary to choose fluid and drop compositions which yield torus formation and breakup in an appropriate distance, generally less than those employed in the laboratory. Surprisingly, this breakup height has received much less attention in the literature. In this script we use existing drop data of Kojima, et al. and the particle cloud data of Pignatel, et al. to determine a simple empirical relationship for predicting how breakup height depends on experimental conditions. This relationship is used to design an experiment which can be conducted in the classroom and its viability and limitations are investigated.

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Kojima Drop Measurements

In these experiments drops of Karo syrup and water were deposited into a solution of Karo syrup and water at a different concentration. Thus, both density difference and viscosity ratio were varied, as well as drop size. While drops were observed to form a torus, expand, and break up, quantitative measurements were only presented for the expansion phase. The characteristic time for expansion would be the ratio of the torus major radius to the expansion rate db/dt. Because the paper presents both the major radius b as well as the torus circle radius eps*b, we can calculate the initial drop volume and Stokes sedimentation time scale based on an undeformed drop. We can also compute the undeformed drop Reynolds number Rec based on the Stokes sedimentation velocity. The Reynolds numbers (calculated in this manner) varied from about .2 to 1, while the dimensionless expansion time varied from about 100 to 200. This non-dimensionalization is chosen so that it aligns with that used by Pignatel for droplets composed of clouds of particles. The data is presented in the same order as Table 2 of that paper.

```
%The fluid viscosity
muf=[.51,.51,.61,.61,.61,.61,.77,.77]';

%The viscosity ratio
lambda = [11.6,7.7,7.1,7.1,4.0,4.0,3.9,2.8]';

%The ring radius
b = [.27,.26,.31,.28,.31,.19,.27,.25]';

%The torus aspect ratio
ep = [.37,.42,.41,.39,.44,.49,.35,.40]';

%We calculate the volume of the torus to get a measure of the original drop
%volume.  This assumes that the ring is symmetric and circular.
v = 2*pi^2*ep.^2.0.*b.^3;

%We have the initial drop radius:
R0 = (v/4/pi*3).^(1/3);

%The measured fluid density
rhof = [1.264,1.264,1.274,1.274,1.274,1.274,1.287,1.287]';

%The measured density difference
drho = [.071,.065,.057,.057,.040,.040,.041,.031]';

g = 980;

%We calculate the Stokes sedimentation velocity:
Us = 2/3*drho*g.*R0.^2./muf./((2+3*lambda)./(1+lambda));

%We calculate the Reynolds number:
Rec = Us.*rhof.*R0./muf

%We have the measured velocity
u = [.93,.90,.95,.75,.82,.42,.36,.32]';

%And the calculated velocities of the tori
ucalc = [0.95 1.0 1.02 0.81 0.88 0.47 0.41 0.34]';

%The measured ring expansion rate
dbdt = [.018,.015,.017,.011,.013,.005,.004,.003]';

%The dimensionless ring expansion time:
te = b./dbdt.*Us./R0
```

```
Rec =

    0.9801
    1.0451
    1.0460
    0.6974
    0.8685
    0.2480
    0.2363
    0.1892
```

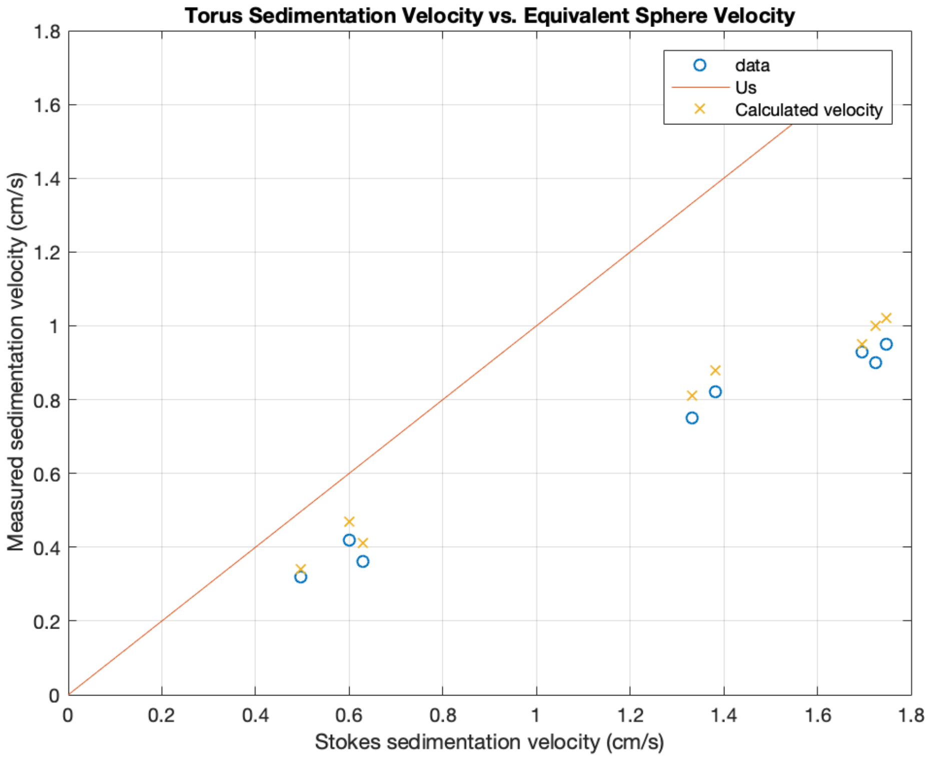
```
te =

  108.9805
  122.2997
  111.0077
  135.3700
  109.6990
  115.1123
  188.8122
  182.2364
```

Sedimentation Velocity

It is interesting to compare the measured sedimentation velocities of the tori to the Stokes sedimentation velocity of a spherical drop of the same volume. Because of the aspect ratio, it would be expected to be smaller, as is observed. The difference between the velocities is due to the aspect ratio of the ring yielding greater drag than a drop of the same volume but of spherical shape as well as a correction due to drop inertia. Kojima calculated the expected velocities using the measured aspect ratio of the tori and the first inertial correction. These are plotted as well, and closely match the measured velocities. The bulk of the correction is due to the aspect ratio of the tori as the inertial correction is small for these conditions. The ratio of the measured velocities to spherical Stokes drop velocities is 0.586 with a sample standard deviation of 0.059.

```
figure(1)
plot(Us,u,'o',[0 max(Us)],[0 max(Us)],Us,ucalc,'x')
xlabel('Stokes sedimentation velocity (cm/s)')
ylabel('Measured sedimentation velocity (cm/s)')
title('Torus Sedimentation Velocity vs. Equivalent Sphere Velocity')
grid on
legend('data','Us','Calculated velocity')

averatio = mean(u./Us)
stdratio = std(u./Us)
```

```
averatio =

    0.5858


stdratio =

    0.0590
```

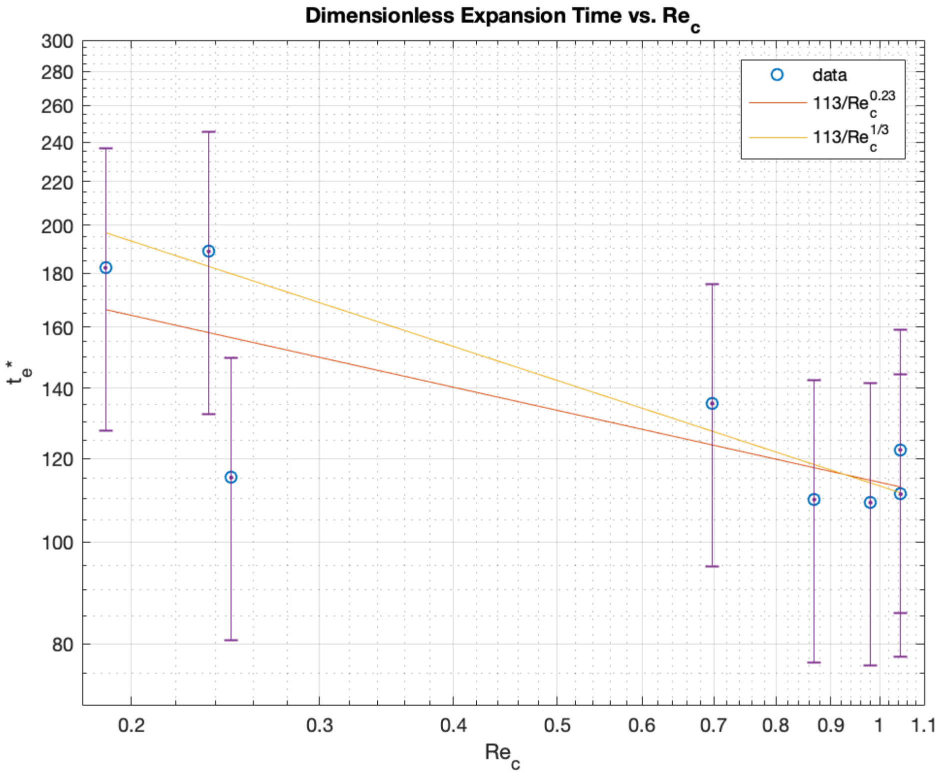


Ring Expansion Time

The dimensionless ring expansion time should be a function of Rec, the characteristic Reynolds number for an undeformed drop. This is plotted below. We fit the data to a power law. Including all the data, the expansion time is a decreasing function of Re, with an exponent of -0.23. If we exclude the outlier (this corresponded to the smallest ring radius and largest value of epsilon - a small, fat torus), then the slope is close to -1/3. Since the sedimentation velocity is proportional to the square of the radius, a -1/3 dependence on Reynolds number would yield a characteristic settling height for ring expansion which is independent of drop volume. The error bars are the 30% uncertainty from the paper by Kojima. Note that there may also be a dependence on the viscosity ratio, however because both viscosity ratio and density difference vary simultaneously, there is no way to test this from the data. The theoretical calculations of Kojima et al., however, suggests that, much like the Stokes sedimentation velocity, the rate of expansion is only a weak function of the viscosity ratio.

```
ak = [ones(size(Rec)),log(Rec)];
x = ak\log(te)

figure(2)
loglog(Rec,te,'o',Rec,exp(ak*x),Rec,113./Rec.^(1/3))
hold on
errorbar(Rec,te,te*0.3,'.')
hold off
axis([.18 1.1 70 300])
xlabel('Re_c')
ylabel('t_e*')
grid on
legend('data','113/Re_c^{0.23}','113/Re_c^{1/3}')
title('Dimensionless Expansion Time vs. Re_c')
```

```
x =

    4.7354
   -0.2273
```



Comparison with Suspension Drops

The data from Pignatel et al for dilute suspensions of particles is given below. This data is for volume fractions of 2% to 10%, and where the drop is in the "macro-inertial" regime: the triangles in figure 12a of that paper. This is the appropriate data to use to compare to a miscible fluid drop, as the other data in figure 12a would be for the "micro-inertial" regime for inertia mediated interactions between individual particles.

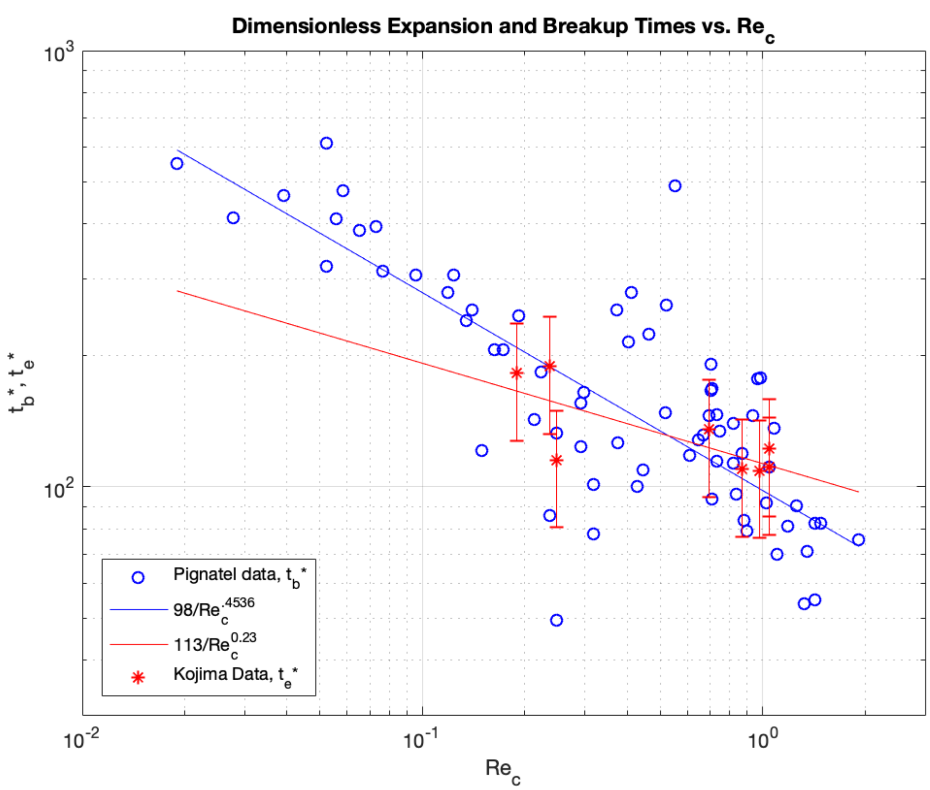
The time presented by Pignatel is the dimensionless breakup time rather than the characteristic ring expansion time, however the two data sets are very similar. Suspension drops were observed to break up when the aspect ratio approached 3 and was defined as the point where the torus developed a bend. The slope of the suspension drop breakup time is slightly greater than that observed for miscible drops, yielding a power law fit of 98/Re^0.45, however it again is quite close to the 1/3 power law yielding heights independent of drop volume.

```
greendata=[0.0522  612.5290
    0.0190  551.2761
    0.0584  476.1021
    0.0390  466.3573
    0.0278  413.4571
    0.0558  410.6729
    0.0731  395.3596
    0.0653  387.0070
    0.0522  320.1856
    0.0764  311.8329
    0.0957  306.2645];

leftreddata=[0.1238  306.4935
    0.1187  279.2208
    0.1403  254.5455
    0.1922  246.7532
    0.1346  240.2597
    0.1625  206.4935
    0.1731  206.4935
    0.2226  183.1169
    0.2984  164.9351
```

```
    0.2922  155.8442
    0.2134  142.8571
    0.2471  132.4675
    0.1495  120.7792
    0.2922  123.3766
    0.3758  125.9740
    0.4444  109.0909
    0.4262  100.0000
    0.3178  101.2987
    0.2370   85.7143
    0.3178   77.9221
    0.2471   49.3506];

toprightreddata=[0.5540  490.1235
    0.4113  279.0123
    0.5220  260.4938
    0.3724  254.3210
    0.4634  223.4568
    0.4032  214.8148
    0.7031  191.3580
    0.9661  176.5432
    0.7031  166.6667];

bottomrightreddata=[1.4266   14.8855
    1.3251   53.8168
    1.4266   54.9618
    1.3498   70.9924
    1.1017   69.8473
    1.9168   75.5725
    1.4803   82.4427
    1.4266   82.4427
    1.1862   81.2977
    0.8993   79.0076
    0.8828   83.5878
    1.0233   91.6031
    1.2537   90.4580
    0.7074   93.8931
    0.8353   96.1832
    1.0424  111.0687
    0.8667  119.0840
    0.7340  114.5038
    0.6103  117.9389
    0.6450  128.2443
    0.6693  131.6794
    0.7477  133.9695
    0.8200  139.6947
    1.0816  136.2595
    0.9331  145.4198
    0.7340  146.5649
    0.6945  145.4198
    0.5169  147.7099
    0.7074  168.3206
    0.9862  177.4809
    0.8200  113.3588];

alldata=[greendata;leftreddata;toprightreddata;bottomrightreddata];

recp = alldata(:,1);
tb = alldata(:,2);

ap=[ones(size(recp)),log(recp)];
xpignatel=ap\log(tb)
```

```
figure(3)
loglog(recp,tb,'ob',recp,98./recp.^(.4536),'b',recp,113./recp.^(0.23),'r')
hold on
loglog(Rec,te,'*r')
errorbar(Rec,te,te*0.3,'.r')
hold off
xlabel('Re_c')
ylabel('t_b*, t_e*')
grid on
axis([.01 3 30 1000])
legend('Pignatel data, t_b*','98/Re_c^{.4536}','113/Re_c^{0.23}','Kojima Data, t_e*',...
    'location','southwest')
title('Dimensionless Expansion and Breakup Times vs. Re_c')
```

```
xpignatel =

    4.5865
   -0.4536
```



Renormalization of Breakup Height

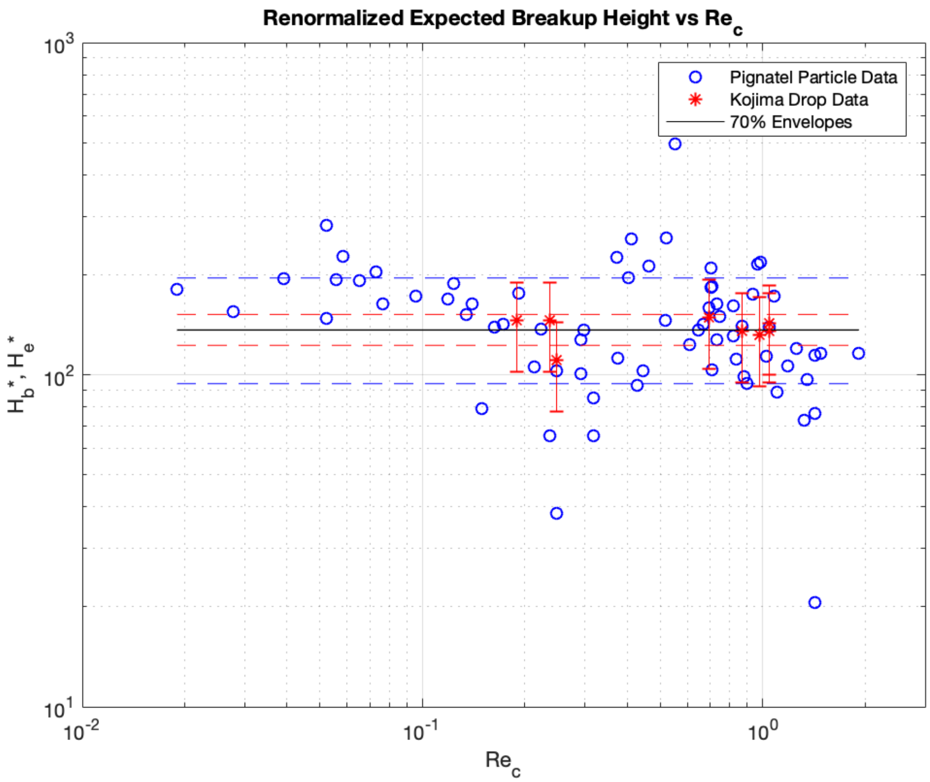
No measurements of breakup height are provided in either paper, however for the Kojima data we can use the measured velocity and characteristic expansion time to estimate a height. Because the velocity reported is that of a torus, however, and much of the time the drop is in its original spherical shape, the average velocity would be greater than this value. From the data in Kojima's paper the average measured torus velocity is 0.586 of the spherical drop velocity. If we take the average sedimentation velocity from initiation to breakup to be the mean of that of a sphere and a torus, then the average velocity would be roughly (1+1/0.586)/2 = 1.353 times the reported values. For the Pignatel data we use the drop Stokes sedimentation velocity and an equivalent correction. The Stokes velocity can be obtained from the reported cloud Reynolds number, and using the correction of 0.793 for the average of the torus and spherical drop velocities yields an estimate of the breakup height.

The Rec^-1/3 dependence of the times suggests that the height should be normalized by fluid properties rather than the drop radius. The appropriate height normalization should be (muf^2/drho*g*rhof)^1/3. This renormalized height is plotted below vs. Rec, and as can be seen is nearly constant (with a lot of scatter) over the whole range of Reynolds numbers investigated in both studies. For both the Pignatel and Kojima data the exp(mean(log(h*))) value is 136 over the whole range of Rec, with a very slight dependence on the Reynolds number. If we assume that the values are independent of Rec, then from the scatter in the log of the data the 95% confidence interval for the mean of the 8 Kojima measurements is [126 148] and that of the 72 Pignatel measurements is [122 152], again very similar. The actual range of breakup times (and corresponding calculated heights) observed by Pignatel varied by over a factor of two - the data was quite scattered. The 70% envelope of observations for both sets is provided as dashed lines in this figure.

```
hestar = 1.353*u.*b./dbdt./(muf.^2.0./drho./980./rhof).^(1/3);
hbstar = 0.793*tb.*recp.^(1/3)*(1.5*5/2)^(1/3); % assumes viscosity ratio of 1

meanhestar=exp(mean(log(hestar)))
stdevloghestar=std(log(hestar))
% confinthestar=meanhestar*exp(stdevloghestar/length(hestar)^.5*tinv([.025 .975],length(hestar)-1))
confinthestar=meanhestar*exp(stdevloghestar*tinv([.15 .85],length(hestar)-1))

meanhbstar=exp(mean(log(hbstar)))
stdevloghbstar=std(log(hbstar))
% confinthbstar=meanhbstar*exp(stdevloghbstar/length(hbstar)^.5*tinv([.025 .975],length(hbstar)-1))
% confinthbstar=meanhbstar*exp(stdevloghbstar*tinv([.15 .85],length(hbstar)-1))
hbsort=sort(hbstar);
confinthbstar=[hbsort(round(72*.15)),hbsort(round(72*.85))]
figure(4)
loglog(recp,hbstar,'ob')
hold on
loglog(Rec,hestar,'*r')
plot([min(recp),max(recp)],meanhestar*[1,1],'k',[min(recp),max(recp)],(ones(2,1)*confinthestar),'r--')
plot([min(recp),max(recp)],meanhbstar*[1,1],'k',[min(recp),max(recp)],(ones(2,1)*confinthbstar),'b--')
errorbar(Rec,hestar,hestar*0.3,'.r')
hold off
xlabel('Re_c')
ylabel('H_b*, H_e*')
grid on
axis([.01 3 10 1000])
legend('Pignatel Particle Data','Kojima Drop Data','70% Envelopes')
title('Renormalized Expected Breakup Height vs Re_c')
```

```
meanhestar =

  136.4399


stdevloghestar =

    0.0957


confinthestar =

  122.5780  151.8694


meanhbstar =

  136.1693


stdevloghbstar =

    0.4555
```

```
confinthbstar =

   93.9552  195.5186
```



Application to the Classroom

From recent publications, it is apparent that many different processes control the ring formation and breakup for falling drops, whether of miscible fluids or suspensions. In particular, suspension drops have been shown to break up via purely Stokesian interactions (due to particle loss and asymmetry), due to "macro-inertial" effects on the length scale of the drop, and due to "micro-inertial" effects on the particle length scale. For fluid drops the breakup is attributed to inertia, however there is also the possibility of transient surface tension effects due to dissimilar materials (necessary to get a density difference). The close agreement between the expansion time of Kojima and the breakup time of Pignatel (where surface tension effects should be absent) make this explanation for the results of Kojima less certain, however. In addition, while other work has demonstrated the existence of such a transient surface tension, it would be expected to play more of a role for small drops rather than large ones (e.g., smaller Re rather than larger Re). This is not apparent from the data, where to get agreement with ring expansion rates surface tensions (chosen as an adjustable parameter) had to be decreased by an order of magnitude for smaller drops of the same fluid pairs.

It is clear (and expected) that the breakup time would be a decreasing function of Re due to the increasing effect of inertia. Why it would have the observed scaling lying between Re^(-.23) and Re^(-.45) is uncertain, however it is usefully approximated by an empirical value of Re^(-1/3) which yields a breakup height roughly independent of drop volume. This slightly underpredicts the time observed by Pignatel at low Re and overpredicts it at higher Re, but falls within the scatter of the data and closely matches that of Kojima. We shall thus use this empiricism in determining optimal fluid/drop combinations. Note that the way in which the drop is introduced also likely affects breakup: the significant inertia of a drop falling into a fluid affects the initial conditions substantially. The original work of Thomson (1885) found that the best rings were produced for drops falling from a height of 1 to 3 inches. In the experiments of Kojima drops were released from a height of 5 cm to yield an oblate spheroid upon impact. In the case of Pignatel, drops were injected directly into the fluid, likely producing a substantially different intial condition.

In order to make a classroom demonstration of the phenomenon, it is necessary to have a reasonably short breakup height. This is particularly true if it is desired to see a cascade of drop breakup events. Thus, to make things work it is necessary to have a reasonably large density difference and low fluid viscosity. Of the two, the dependence on fluid viscosity is somewhat greater, however too low a fluid viscosity (or too high a density difference) would lead to velocities and Reynolds numbers well beyond the conditions explored by

Kojima or Pignatel. Too short of a breakup height is also undesirable as it would be difficult to see and would be more susceptible to variations in the initial conditions.

For a demonstration in a graduated cylinder, the behavior is further complicated by wall reflections. The diameter of a 500ml cylinder, for example, is about 4.6cm inside diameter, thus for a 1 cm diameter drop the aspect ratio would be less than 1:5 even before expansion into a torus. This would reduce the sedimentation velocity, but also may limit ring expansion and instability. It also introduces another length scale into the problem, and would certainly lead to variations of breakup height for different volume drops.

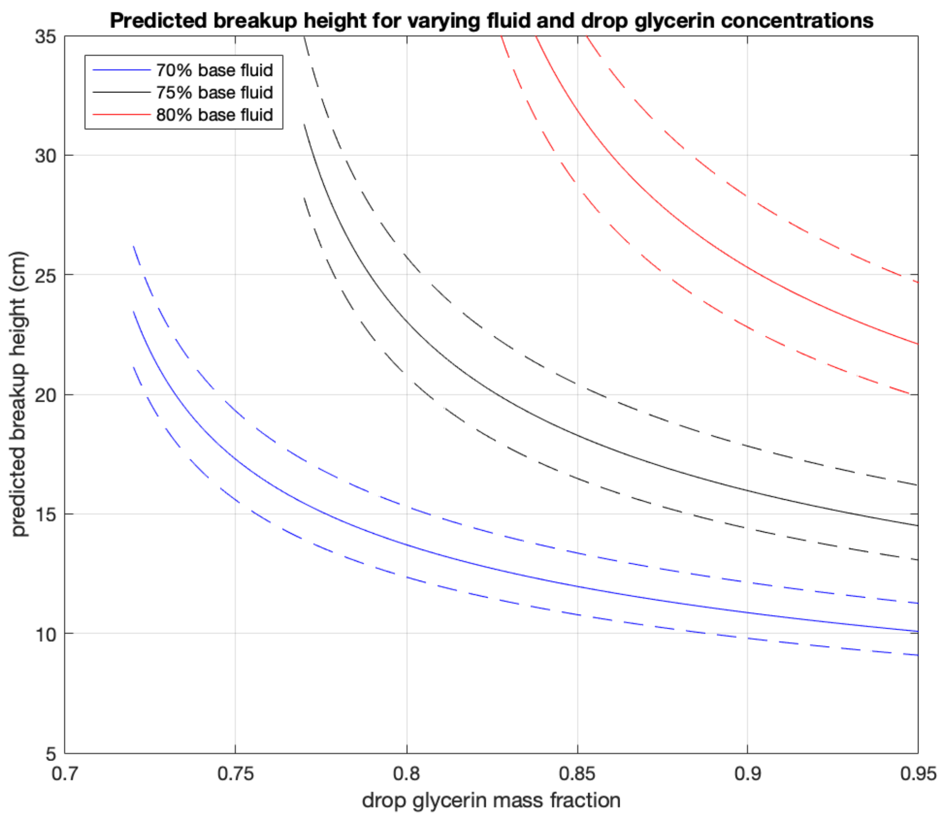
A convenient mixture is glycerin/water solutions of different concentrations. The expected breakup height for several glycerin/water solutions and drop compositions is plotted below. The dashed lines are indicative of the width of the envelope containing 70% of the observed breakup times from the observations of Pignatel. It would be expected that drop breakup observations in glycerin/water mixtures would be similarly scattered.

It would be desirable to have a breakup height great enough so that the transition is clearly visible, yet short enough that a cascade of breakups is also visible. An attractive fluid is a 75% by mass glycerin/water solution yielding a viscosity of 0.28 poise. The concentration of the drop phase would need to be higher, up to about 95% for a viscosity ratio of 10. Predicted breakup heights based on this combination range from 15 to 30 cm. A fluid composition of 75wt% and a droplet composition of 85wt% would be a good combination, yielding a viscosity ratio of 2.8 and a density difference of 0.027 g/cm^3, with a predicted breakup height of 18 cm. This is significantly less than the height of about 35cm achievable in a 500ml graduated cylinder. In contrast, if a somewhat higher glycerin composition for the fluid is used (e.g., a mass fraction of 0.85, yielding a viscosity of 0.79 poise), then the predicted breakup height would exceed the height of a 500ml cylinder for all drop glycerin concentrations. A lower base fluid glycerin concentration of 65wt% (viscosity of 0.12 poise) would yield very short breakup heights and times, to the point where visualization of transitions would be difficult to distinguish from drop entrance effects.

For this choice of fluid and drop compositions, a 0.1 ml drop would have a diameter of 0.58 cm, a spherical drop Stokes velocity of 1.9 cm/s, and a Rec of 2.5. The latter is somewhat greater than the values explored by Kojima, and just a bit larger than suspension drops (in the macro-inertial range) examined by Pignatel. It would be expected to break up in about 12 seconds, all reasonable values for a demonstration. A 500ml graduated cylinder is of sufficient height to observe a secondary breakup cascade as well. A smaller drop would put it in the same Rec range as those of Kojima, who used drops as small as 0.03 ml, however too small a drop becomes both difficult to see in a demonstration and to administer in a controlled manner without more extensive equipment than dripping from a tube. More viscous base fluids such as employed by Kojima would reduce the Reynolds number as well, but at the cost of increasing the height required for breakup.

```
cdlim = .95;
cfall = [.70 .75 .80 .85];
cf = cfall(1);
cd = [cf+.02:.001:cdlim]';

hpred = 136*(viscosity(cf)^2./((density(cd)-density(cf))*g*density(cf))).^(1/3);
figure(5)
p1=plot(cd,hpred,'b');
xlabel('drop glycerin mass fraction')
ylabel('predicted breakup height (cm)')
title('Predicted breakup height for varying fluid and drop glycerin concentrations')
grid on

cf = cfall(2);
cd = [cf+.02:.001:cdlim]';

hpred = 136*(viscosity(cf)^2./((density(cd)-density(cf))*g*density(cf))).^(1/3);
hold on
p2=plot(cd,hpred,'k');
hold off

cf = cfall(3);
cd = [cf+.02:.001:cdlim]';

hpred = 136*(viscosity(cf)^2./((density(cd)-density(cf))*g*density(cf))).^(1/3);
hold on
p3=plot(cd,hpred,'r');
```

```
cf = cfall(1);
cd = [cf+.02:.001:cdlim]';

hpred = (viscosity(cf)^2./((density(cd)-density(cf))*g*density(cf))).^(1/3)*confinthestar;
figure(5)
hold on
plot(cd,hpred,'b--')
hold off

cf = cfall(2);
cd = [cf+.02:.001:cdlim]';

hpred = (viscosity(cf)^2./((density(cd)-density(cf))*g*density(cf))).^(1/3)*confinthestar;
hold on
plot(cd,hpred,'k--')
hold off

cf = cfall(3);
cd = [cf+.02:.001:cdlim]';

hpred = (viscosity(cf)^2./((density(cd)-density(cf))*g*density(cf))).^(1/3)*confinthestar;
hold on
plot(cd,hpred,'r--')
hold off

axis([.7 .95 5 35])

subset=[p1;p2;p3];
legend(subset,[num2str(cfall(1)*100),'% base fluid'],...
    [num2str(cfall(2)*100),'% base fluid'],...
    [num2str(cfall(3)*100),'% base fluid'],...
'Location','northwest')
```



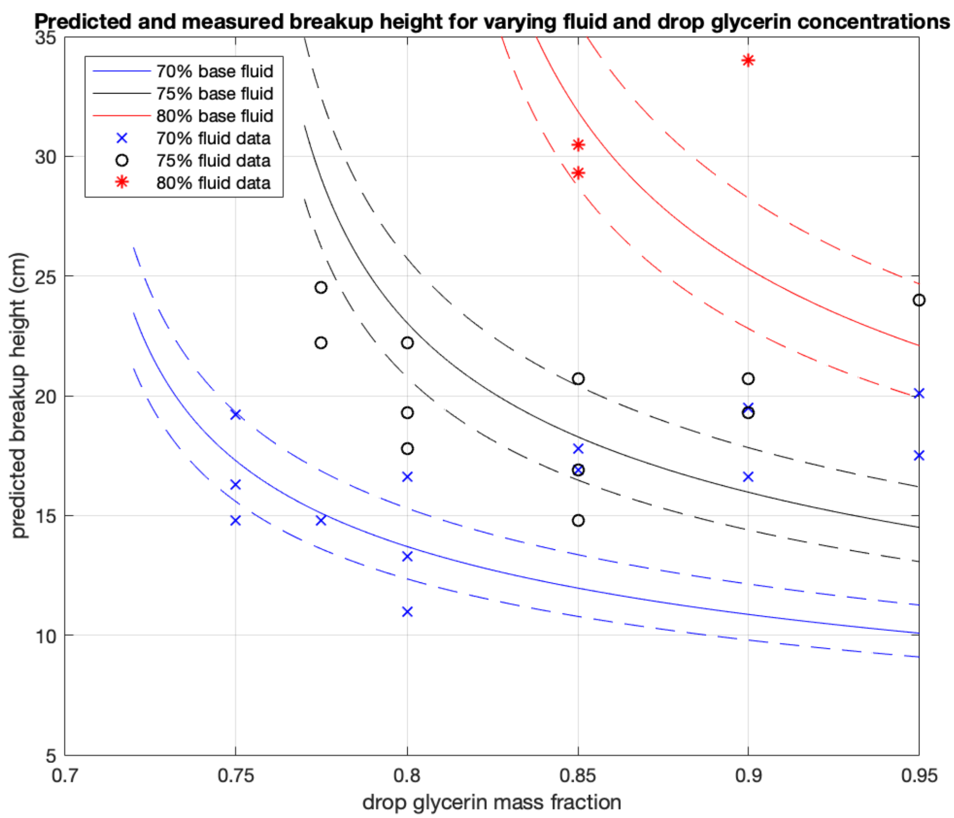
Experiment

In order to confirm the application of the above calculations to classroom demonstrations, experiments were performed dripping glycerin/water solutions into a 500ml graduated cylinder (diameter of 4.6cm) from a 1 ml syringe barrel which had its tip cut off. Volumes varied, however were typically between 0.05ml to 0.15ml. The syringe was held 3cm above the fluid surface yielding an impact velocity of approximately 8 cm/s, and the total filled column depth was 34 cm. The onset of breakup was taken to be the point where the expanding torus developed an observable bend (in agreement with the criteria of Pignatel). Typically, complete breakup occurred within a few centimeters of that point, usually yielding two droplets which would further break up in a cascade.

The results of these experiments are depicted below. As can be seen, for drop compositions close to that of the fluid in the column there is reasonable agreement with the empirical formula given above. The initial decrease in breakup height with increasing drop glycerin concentration is due to the larger density difference yielding a higher velocity and Reynolds number, as expected. For larger drop concentrations, however, the breakup height was observed to increase again. This increase is likely due to the increased drop viscosity ratio (greater than 3 where the breakup height increases) counteracting the increased density difference. The work of Kojima et al showed that the ring expansion rate was insensitive to the viscosity ratio. For the more viscous of the drops employed here it was observed that the rings would expand, however the Rayleigh-Taylor instability was delayed, resulting in tori which were noticeably thinner at breakup. The Rayleigh-Taylor instability requires draining of the drop fluid along the ring rather than just uniform expansion, and thus is more likely to be sensitive to the drop viscosity. For the measurements of Pignatel the viscosity ratio was always close to unity.

```
% 75% base fluid data
dropconc75 = [.775 .775 .85 .85 .85 .8 .8 .8 .9 .9 .95];
height75 = [24.5 22.2 14.8 16.9 20.7 17.8 22.2 19.3 19.3 20.7 24];

% 80% base fluid data
dropconc80 = [.85 .85 .90];
height80 = [29.3 30.5 34];

% 70% base fluid data
dropconc70 = [.75 .75 .775 .775 .8 .8 .85 .85 .9 .9 .95 .95 .75 .8];
height70 = [14.8 16.3 14.8 14.8 11 13.3 16.9 17.8 19.5 16.6 17.5 20.1 19.2 16.6];
```

```
figure(5)
hold on
d1 = plot(dropconc70,height70,'xb');
d2 = plot(dropconc75,height75,'ok');
d3 = plot(dropconc80,height80,'*r');
hold off
subset=[p1;p2;p3;d1;d2;d3];
legend(subset,[num2str(cfall(1)*100),'% base fluid'],...
    [num2str(cfall(2)*100),'% base fluid'],...
    [num2str(cfall(3)*100),'% base fluid'],...
    '70% fluid data','75% fluid data','80% fluid data','Location','northwest')
title('Predicted and measured breakup height for varying fluid and drop glycerin concentrations')
```



Conclusion

The empirical formula presented here was useful in developing an effective in-class demonstration of the drop breakup phenomena. When employed, the students were found to be very engaged, gathering closely around it (images were also projected on a the classroom screen) and trying it themselves. While the one-third power dependence on Reynolds number is puzzling (and indeed is likely to occur only over a limited range), the dependence on the parameters presents a useful example of using dimensional analysis to collapse and analyze data. The normalization which was found useful would arise naturally from the assumption that the density of the droplet fluid only comes into the problem through the buoyancy term drho*g and from the empirical observation that breakup height is insensitive to the droplet volume and viscosity. Thus, this analysis and demonstration is now attached to the class in which dimensional analysis is discussed.

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